Crowding and the Moments of Momentum

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Abstract

We develop a model of crowding as in Stein (2009), applied to momentum. The model demonstrates how anticipated and unanticipated crowding translate into the first three moments of momentum returns. We test the model's predictions using proxies for momentum crowding developed from 13F data and find that unanticipated crowding predicts negatively momentum returns, particularly as it relates to the number of institutions following a momentum strategy. By contrast, the intensity of momentum trade is less of a factor. Crowding is also negatively related to volatility of momentum returns, consistent with forward looking investors anticipating, and avoiding, strategy risk. Finally, unanticipated crowding predicts large declines (fat left tails) in momentum profits, consistent with the feedback effects outlined in Stein (2009).

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1 Introduction

If informed investors are capital constrained and risk averse, they can be expected to leave some of their information on the table, pushing winner (loser) stock prices up (down) only part way toward what they perceive to be fundamental value. Momentum investing can be regarded as an effort to rationally infer this residual information and act on it by extrapolating past differential returns on winner and loser stocks into a signal of as-yet-unpriced deviations in fundamental value. While this strategy further drives prices toward fundamental value, a momentum factor premium will remain if momentum investors are themselves capital constrained and risk averse.

We use this logic to develop a model of momentum factor returns based on the distribution of capital across three investor types: informed investors, momentum investors, and counterparty investors. Informed investors observe heterogeneous, noisy signals of fundamental value for a multitude of stocks. Momentum investors observe no private signals, but they maintain rational expectations regarding the price formation process. Counterparty investors take the other side of both informed and momentum investors' trades, by definition. Our model is closely related to the Stein (2009) model of crowding applied to under-reacting newswatchers. However, the focus here is how capital uncertainty affects return moments rather than equilibrium pricing and efficiency considerations as in Stein (2009). Thus, the model is a more specific development steered towards framing the empirical analysis.

The existence of counterparty investors is a concern [e.g., Milgrom and Stokey, 1982] that we, like most studies, do not directly address. We simply assume a random quantity of counterparty capital that willingly accepts investment exposure in proportion to the price concession it receives relative to a public-information valuation. We take the required price concession to be an exogenous parameter meant to reflect both partial aversion to adverse selection and the option to not participate. At extreme values of this parameter, the no-trade theorem obtains.[1]

The momentum cycle begins with informed investors privately observing heterogeneous signals of fundamental value for a number of stocks. They then enter a call auction for each stock on

[1]Many studies address the irrationality of counterparty trading by assuming it satisfies some unstated exogenous motive that dominates adverse selection concerns (i.e. noise trading). In the resulting equilibrium, informed traders find willing counterparties according to a price-impact schedule parameterized by the variance of noise trading demands. Our approach is essentially a re-parameterization that sidesteps the complexity of a noisy rational expectations framework. The cost is explicit incorporation of irrationality. The benefit is a greatly streamlined model that gets to the same place.

the rank date, and seek to trade in proportion to the strength of their private signals. Momentum and counterparty investors join that auction, and all investors condition on the market-clearing price. When this rank-date market clears, the momentum portfolio is identified, as is its ranking-period return. An evaluation period then begins, and ends with all investors observing a public release of fundamental value. At that point information is again symmetric and the momentum cycle is completed.

A greater quantity of momentum capital applied in the rank-date market should result in a lower momentum premium, since the residual information of informed capital is then diluted across more momentum capital. That is a straightforward implication of supply and demand considerations. As in Stein (2009), the more important result pertains to the role of uncertainty in momentum capital. Our development emphasizes the nonlinear nature of unanticipated momentum capital's impact on the return characteristics of momentum, particular the third moment (negative-tail returns).

In a framing similar to Stein (2009), we assume tht momentum investors cannot distinguish between abnormally large ranking-period returns that are driven by informed investors' private signals, and abnormally large ranking-period returns that are driven by unanticipated crowding by other momentum investors. They condition on a belief about the relative contribution from each source (information versus crowding), and form demands accordingly. As a result of the ambiguity in the source of ranking-period price moves, unanticipated crowding induces positive feedback trading.

We conjecture that it is difficult to forecast the number of investors who enter into or exit from a momentum strategy, but that it is relatively easy to forecast the intensity of their trading conditional on having adopted the strategy This conjecture is based on the reasoning that the trade intensity of a representative momentum trader reflects a relatively homogenous reaction to a common setting, as formulated in the model. In that case the number of momentum investors acting at a given time should be the primary source of crowding uncertainty. Alternatively, if assets under management (AUM) is a strong determinant of trading intensity as would be the case under CRRA prefernces, it may be that the intensity of capital allocation is also an important source of uncertainty as AUM can differ susbtantially across investors.

Our empirical specification mirrors this decomposition of uncertainty. Using 13 F data we form proxies for momentum capital by characterizing each institution according to whether they seem to follow a momentum strategy or not. Thus, the basic variable is an indicator function, which is then summed across institutions to proxy for the count of momentum participants. Investors know their own type at the time of trade, but not this aggregate count. We generically refer to this as the *count* proxy.

Momentum capital is the count of momentum investors times the magnitude of a representative momentum trade. Our second empirical proxy, which we here generically refer to as the *capital* proxy, is essentially the *count* proxy times this representative trade intensity. In the model this intensity multiplier reflects an optimal response to current conditions (risk tolerance, information environment, etc), implying different predictions for the *capital* proxy regarding the moments of momentum returns. First, because the representative intensity is presumed known to each optimizing momentum investors, it does not generate the same feedback effects identified by Stein (2009). Second, trading intensity is generally related positively to the momentum return residual to market clearing on the rank date, reflecting optimal anticipatory demands. Thus, the *capital* proxy may have different implications for predicting moments of momentum return than the *count* proxy, and it is the latter that should be most associated with feedback effects from crowding.

To capture anticipated versus unanticipated crowding, we use both a levels and a changes specification in the *count* and *capital* proxies. We also consider a third empirical specification to proxy the uncertainty in unanticipated crowding, using the expected volatility from a GARCH(1, 1) specification of each of the *count* and *capital* proxies. On the returns side, we consider raw momentum returns as well as the abnormal return on momentum conditioning on the Fama-French three factors and on a dynamic version of the Fama-french model. Finally, to analyze the impact of crowding on the various moments of momentum we consider the level of momentum returns, the volatility, and the probability of a left-tail (extreme negative return) event.

We find that unanticipated crowding predicts momentum returns, as hypothesized in the model and proxied with the change in crowding. First, consistent with *count* being the primary source of unanticipated momentum capital. we find that changes in the *count* proxy best predict negatively momentum returns. Generally, we do not find the same negative predictive relation for momentum returns result with the level of *count*, or using changes (or levels) in the *capital* proxy.

Second, and most central to the identification of feedbacking effects from crowding, probit regressions identify a statistically significant positive relation between changes in the *count* proxy and the probability of an extreme negative momentum return. This is again consistent with the number of competing institutions engaging in a momentum strategy being the primary source of uncertainty in crowding, and uncertainty in crowding generating feedback effects. More importantly, this is confirming evidence for the central role of feedbacking effects from crowding.

Third, momentum investors appear to anticipate – and shun – risk in the momentum strategy. We find that changes in *count* predict negatively momentum return volatility. That is, the number of investors adopting the momentum strategy appears to forecast high risk in that strategy. We also find that the estimated uncertainty (i.e., Garch volatility) in the crowding measures forecasts momentum return volatility.[2] Finally, consistent with the notion that momentum investors' aversion to crowding risks derives from an anticipated impact on the risk of the strategy, we find that volatility in crowding positively predicts momentum returns indicating compensation for this risk.

The study is outlined as follows. Section 2 provides a brief summary of the existing literature to put the study in context. Section 3 provide the development and results of the model and section 4 explores its predictions. Section 5 concludes the study.

2 Existing literature

Our paper is related to the empirical and theoretical literature on momentum. Momentum was initially documented for US stock returns [Levy, 1967, Jegadeesh and Titman, 1993] and has since been documented for stock returns in most countries [Rouwenhorst, 1998] and across asset classes [Asness et al., 2013]. Besides its very high average returns, momentum carries significant downside risk or negative skewness in the form of occasional large crashes [Barroso and Santa-Clara, 2015, Daniel and Moskowitz, 2016]. Existing research also shows that institutional investors are momentum traders, i.e., tilt their portfolios towards momentum stocks [Grinblatt et al., 1995, Lewellen, 2011, Edelen et al., 2016]. Our aim is to investigate the extent to which uncertainty regarding institutional participation in the momentum strategy is a plausible source of these return characteristics.

The related theoretical literature on momentum offers theories based on institutional investors and fund flows [Vayanos and Woolley, 2013] or behavioral biases such as over-reaction / self-

[2] A control for lagged volatility in momentum returns is included in this and all regressions.

attribution [see, e.g., Daniel et al., 1998, Barberis et al., 1998] or the gradual diffusion of information among investors [see, e.g., Hong and Stein, 1999, Andrei and Cujean, 2017]. Our model differs in its delineation of a critical role for momentum capital - particularly unexpected momentum capital. In this sense it is closest to Stein [2009], which develops the theoretical foundation for a destabilizing influence from crowding. In his model arbitrageurs try to exploit the under-reaction of naïve investors but face uncertainty regarding the total amount of arbitrage capital. Stein shows how the inability of arbitrageurs to know in real time how many others are following the same strategy creates a coordination problem. As a result, rather than providing pricing improvement, it can be that arbitrage capital pushes prices further away from fundamentals. As discussed in the introduction, our model builds off of this insight from Stein [2009], with a development focused more specifically on momentum and a framing of empirical analyses.

Kondor and Zawadowski [2015] study whether the presence of more arbitrageurs improves welfare in a model of capital reallocation. Trades in the model can become crowded due to imperfect information, but arbitrageurs can also devote resources to learn about the number of earlier entrants. They find that if the number of arbitrageurs is high enough, more arbitrageurs do not change capital allocations, but decrease welfare due to costly learning. Abreu and Brunnermeier [2002] argue that arbitrage may be limited and delayed due to synchronization risk, i.e., the uncertainty about when the other arbitrageurs will trade.

Related empirical research includes Hanson and Sunderam [2014] who construct a measure of the capital allocated to momentum and the valuation anomaly (book-to-market or B/M) using short-interest. They find some evidence that an increase in arbitrage capital has reduced the returns on B/M and momentum strategies. In addition, Lou and Polk [2013] proxy for momentum capital with the residual return correlations in the short and long leg of the momentum strategy and find that momentum profits are higher in times of lower momentum capital. Our evidence supports their finding, wth a different approach and insights in proxying momentum capital.

3 Model

Section 3.1 lays out the assumptions and setting of the model. Section 3.2 develops the demands of each investor type, and section 3.3 develops the equilibrium on the rank date. The predictions from this equilibrium are then developed in section 3.4.

3.1 Setting

Stocks are indexed by j. Each pays a discrete dividend $X_{j,t}$. Dividends evolve according to

$$\log\left(X_{j,t+1}/X_{j,t}\right) = \chi_{t+1} + \iota_{j,t+1}\frac{\delta_{t+1}}{2}, \tag{1}$$

where χ_{t+1} is a random innovation common to all stocks with variance σ_χ^2 that generates the market return; the indicator $\iota_{j,t+1}$ selects the momentum portfolio, taking on the value 1 or -1 for 10% of all stocks (in each leg); and δ_{t+1} generates the differential return on the two groups of stocks, with variance σ_δ^2 and mean $d - \sigma_\delta^2/2$. We do not model an idiosyncratic component to δ_{t+1}, as diversification in implementing a momentum strategy would eliminate its relevance. All investors know σ_δ^2 and σ_χ^2. We refer to stocks with $\iota_{j,t+1} = 1\,(-1)$ as long leg (short leg) stocks.

At the beginning of the momentum cycle (time t) information is symmetric, hence each investor rebalances their holdings to the market portfolio. This results in a public-information valuation vector $\boldsymbol{P}_t = \boldsymbol{X}_t/r$ where r denotes the required return on the market portfolio. We do not model r. At some intermediate time that we refer to as the rank date (indexed '$t + Rnk$' $< t + 1$), a subset of 'informed' investors observes d_{t+1} and $\boldsymbol{\iota}_{t+1}$ and trades at market-clearing prices $\boldsymbol{P}_{t+Rnk}$. This generates the ranking period return $\boldsymbol{r}_{t\to t+Rnk}$ where $r_{j,t\to t+Rnk} = \log\left(P_{j,t+Rnk}/P_{j,t}\right)$. Information symmetry is regained at time $t+1$ when $\boldsymbol{X}_{t+1}$ is publicly revealed and all investors rebalance to the market portfolio. Because of this reset, we drop the t subscripts and focus on just one momentum cycle, using the subscript $_0$ to denote time t values and the subscript $_1$ for time $t+1$ values.

$r_{j,0\to Rnk} = 0$ for neutral stocks ($\iota_{j,1} = 0$) because all investors maintain belief $E\left[X_{j,1}\right] = X_{j,0}$ for such stocks. Because each informed investor has homogeneous expectations of dividend growth for long-leg stocks, $E\left[X_{j,1}\right]/X_{j,0} = e^{d_1/2}$, and each uninformed (i.e., momentum and counterparty) investor likewise has homogenous expectations for long-leg (indeed, all) stocks, $E\left[X_{j,1}\right]/X_{j,0} = 0$, each long leg stock expriences the same ranking period return. Denote this return $r_{+,0\to Rnk}$. Short-leg stocks likewise have a homogenous expected return, of the same magnitude and opposite sign. Denote this return $r_{-,0\to Rnk}$. Thus, we can compress the returns of individual stocks to a return on the momentum portfolio. Let m denote this return. In the ranking period

$$m_{0\to Rnk} = \sum_{\iota_j=+1} \frac{X_{j,0}}{X_{\pm,0}} r_{+,0\to Rnk} - \sum_{\iota_j=-1} \frac{X_{j,0}}{X_{\pm,0}} r_{-,0\to Rnk} = r_{+,0\to Rnk} - r_{-,0\to Rnk}, \tag{2}$$

where stocks are weighted by their beginning of ranking-period valuation. Evaluation-period return on the momentum factor are

$$m_{Rnk\to 1} = d_1 - m_{0\to Rnk}, \tag{3}$$

which follows from

$$\begin{array}{llll} \text{long leg:} & e^{r_{+,0\to Rnk}} e^{m_{+,Rnk\to 1}} = e^{d_1/2} & \Rightarrow & m_{+,Rnk\to 1} = \dfrac{d_1}{2} - r_{+,0\to Rnk} \\ \text{short leg:} & e^{r_{-,0\to Rnk}} e^{m_{-,Rnk\to 1}} = e^{-d_1/2} & \Rightarrow & m_{-,Rnk\to 1} = -\dfrac{d_1}{2} - r_{-,0\to Rnk}. \end{array}$$

Thus, we seek an expression for $m_{Rnk\to 1}$ by way of $m_{0\to Rnk}$.

Ranking-period returns are determined by equating the demands of three investor groups. Two are proactive, responding to information signals with active trading. The third is reactive, supplying the positions demanded by proactive investors.

Informed investors control beginning-of-cycle capital K_I. They privately observe $\boldsymbol{\iota}_{t+1}$ and d_1 and initiate the momentum portfolio on the rank date by trading on that information. Each forms perfect-information rank-date expectations for the momentum factor return

$$E_I\left[m_{Rnk\to 1} \middle| d_1, m_{0\to Rnk}\right] = d_1 - m_{0\to Rnk}. \tag{4}$$

Momentum investors control beginning-of-cycle capital K_M. They do not observe d_1 or ι_1, but condition their demands on $m_{0\to Rnk}$

$$E_M\left[m_{Rnk\to 1} \middle| m_{0\to Rnk}\ ,\quad \cdot\ \right] = E_M d_1 - m_{0\to Rnk}. \tag{5}$$

Counterparty investors trade counter to informed and momentum investors and are therefore adverse selected. A fraction $(1 - L)$ recognize this, exercising their no-trade option by maintaining $E_C m_{Rnk\to 1} = 0$. The remaining fraction L face an exogenous motive for trade and willingly accept adverse selection as a cost of sastisfying that motive. The de facto expectations of a counter-party investor are

$$E_C m_{Rnk\to 1} = (1-L)\cdot 0 + L\cdot(-m_{0\to Rnk}) = -L\cdot m_{0\to Rnk}. \tag{6}$$

Counterparty investors control beginning-of-cycle capital K_C.

All investors hold power utility preferences with relative risk aversion ρ, maximizing [3]

$$\log E\left[u\left(K_1\right)\right] = \log E\left[\frac{K_1^{1-\rho}}{1-\rho}\right] \tag{7}$$

on the rank date, where K_1 denotes the investor's capital at the end of the momentum cycle. We use the methodology of Campbell and Viceira [2002, appendix] to derive the following expression for demands:

$$Demand = \frac{Em_{Rnk\to 1}}{\rho\sigma_\delta^2} K_0, \tag{8}$$

where K_0 denotes capital at the beginning of the momentum cycle. Details are in the appendix.

3.2 Rank-date pricing of the momentum portfolio

Using each investor type's expectations in (8), the market clearing condition is

$$L\frac{m_{0\to Rnk}}{\rho\sigma_\delta^2}\cdot\tilde{K}_C = \frac{\tilde{d}_1 - m_{0\to Rnk}}{\rho\sigma_\delta^2}\cdot K_I + \frac{E_M\tilde{d}_1 - m_{0\to Rnk}}{\rho\sigma_\delta^2}\cdot\tilde{K}_M. \tag{9}$$

Using $\tilde{k}_{type}$ to denote the $fraction$ of capital from the subscripted type, market-clearing implies

$$\tilde{m}_{0\to Rnk} = \tilde{d}_1\cdot k_I + E_M\tilde{d}_1\cdot\tilde{k}_M. \tag{10}$$

Stein (2009) proposes a linear specification for the beliefs of momentum investors. Let us suppose that they follow this conjecture, i.e., that ranking period returns follow

$$\tilde{m}_{0\to Rnk} = \beta\cdot\tilde{d}_1. \tag{11}$$

[3] We assume that $\rho > 1$ throughout the paper.

More specifically, suppose that momentum investors conjecture

$$\beta \equiv E_M\left(\tilde{k}_I + \tilde{k}_M\right). \tag{12}$$

That is, the fraction of informed traders' beliefs that gets incorporated into ranking period returns is equal to the fraction of capital that trades with a rational information motive. Let $\tilde{\beta}$ denote the coefficient that results from this conjecture. Using (11) and (12) in (10) and a bit of algebra,

$$\tilde{\beta} = \beta\frac{1+\tilde{\nu}_I}{1-\tilde{\nu}_M}, \tag{13}$$

where

$$\tilde{\nu}_I = \frac{\tilde{k}_I - E\tilde{k}_I}{Ek_I} \tag{14}$$

$$\tilde{\nu}_M = \frac{\tilde{k}_M - E\tilde{k}_M}{Ek_I} \tag{15}$$

denotes unanticipated informed and momentum capital, both normalized by the expected informed capital. To focus on the effects of crowding uncertainty, we simplify by presuming $\tilde{k}_I \equiv E\tilde{k}_I$ or $\nu_I = 0$. From (11) and (13) with this simplification

$$E_M\tilde{d}_1|\tilde{m}_{0\to Rnk} = E_M\tilde{\beta}^{-1}\tilde{m}_{0\to Rnk} = \beta^{-1}E_M\left(1-\tilde{\nu}_M\right)\tilde{m}_{0\to Rnk} = \beta^{-1}\tilde{m}_{0\to Rnk}, \tag{16}$$

as conjectured (thus confirming rational expectations given linear beliefs). Thus,

$$\tilde{m}_{0\to Rnk} = \tilde{d}_1\cdot\beta\frac{1}{1-\tilde{\nu}_M}. \tag{17}$$

Momentum factor returns from the rank date to the end of the momentum cycle are

$$\tilde{m}_{Rnk\to 1} = \tilde{d}_1\left(1-\beta\frac{1}{1-\tilde{\nu}_M}\right). \tag{18}$$

3.3 Model results

Let us first focus on the β term in (18), holding $\tilde{d}$ constant and for the moment ignoring unanticipated momentum capital (i.e., assume $\tilde{\nu}_M = 0$). β is the fraction of the information of informed traders that is incorporated into ranking period return. A higher value of expected momentum capital, $E\tilde{k}_M$, implies a greater degree to which $\tilde{d}$ is incorporated into ranking period returns, which in turn implies a lower residual remaining in the form of a momentum return. We summarize this as

Result 1 *The diffusion of information into ranking period returns is increasing in the expected momentum capital, $E\tilde{k}_M$, implying a negtive relation between $E\tilde{k}_M$ and momentum returns, where*

$$\left.\frac{\partial \tilde{m}_{Rnk\to 1}}{\partial E\tilde{k}_M}\right|_{\tilde{d}_1 fixed, \tilde{v}_M = 0} = \tilde{d}_1. \tag{19}$$

That is, an increase in anticipated momentum capital causes a constant proportional decrease in momentum returns.

Now consider the effects of $\tilde{v}_M$ on momentum returns. To explore this term, first recall that momentum investors form beliefs of end-of-cycle dividends by extrapolating ranking period returns under the conjecture (11). Using expression (17) for the $\tilde{m}_{0\to Rnk}$ that results from this conjecture, momentum investors' beliefs are

$$E_M\tilde{d}_1 = \beta^{-1}\tilde{m}_{0\to Rnk} = \tilde{d}_1 \cdot \frac{1}{1-\tilde{v}_M}. \tag{20}$$

That is, the $\tilde{d}_1$ that momentum investors infer from ranking period returns is distorted by unanticiated crowding, $\tilde{v}_M$. The larger $\tilde{v}_M$, the greater the distortion. Potentially, this distortion is unbounded as $\tilde{v}_M$ goes to 1. Now observe that, from the demand expression (8), the aggressiveness with which each dollar of momentum capital is allocated to the momentum trade is determined by the intensity of beliefs. Putting the two observations together, we see that unanticipated momentum capital $\tilde{v}_M$ causes *each dollar* of momentum capital to be allocated more aggressively. This generates a positive feedback effect that amplifies the assimilation of $\tilde{d}_1$ into ranking period returns and attenuates the residual momentum return.

Result 2 *Unanticipated moment capital $\tilde{v}_M$ is negatively related to momentum returns by way of the distorting effect that $\tilde{v}_M$ has on inferences of $\tilde{d}_1$ from ranking period returns:*

$$\left.\frac{\partial \tilde{m}_{0\to Rnk}}{\partial \tilde{v}_M}\right|_{\tilde{d}_1, \beta} = \frac{\tilde{d}_1\beta}{(1-\tilde{v}_M)^2}. \tag{21}$$

Because this becomes increasingly large at high values of $\tilde{\nu}_M$, the negative effect of $\tilde{\nu}_M$ on momentum returns is potentially unbounded implying an elevated probability of large negative momentum returns (fat left tails). In particular, when $\tilde{\nu}_M$ exceeds the anticipated supply of counterparty capital

$$\tilde{\nu}_M > 1 - E_M\left(\tilde{k}_I + \tilde{k}_M\right) = L \cdot E_M\tilde{k}_C \tag{22}$$

momentum investors extrapolate ranking period returns past fundamental value. At that point, momentum trading becomes feedback trading resonating with itself. By contrast, at low (negative) values of $\tilde{\nu}_M$, the distortion effect attenuates so the right tail in the distributon of momentum returns inherits the distribution of $\tilde{d}_1$.

Result 2 outlines three hypotheses: (1) unanticipated momentum capital predicts negatively momentum returns; (2) the effect of unanticipated momentum capital is potentially stronger than that of anticipated momentum capital (which is constant and bounded); and (3) unanticipated momentum capital predicts a fat left tail in the distribution of momentum returns. In the next section we explore these predictions as well characterize the relation between momentum capital and returns more generally.

4 Empirical section

We base our empirical analysis on the trading of institutions. Because institutions dominate equities trading, we seek empirical proxies that retain their meaning even if institutions trade only with themselves. Thus, the empirical analog to the quantity of momentum capital in quarter t, $\tilde{k}_M$, should refer to an ex ante measures of capital committed to momentum during that quarter, rather than to the aggregate realized institutional purchase/sale of momentum stocks. In the model we have framed $\tilde{k}_M$ as a random number of institutions engaging in momentum, each of whom trades with a deterministic demand curve. We set up our empirical proxies likewise, considering crowding measures based on both the count of institutions following a momentum strategy (which we think of as the most plausible source of unanticipated crowding), and on the amount of capital backing up that strategy (which we think of as a more deterministic overlay in intensity of demands related to, for example, risk tolerance). Of course, it is certainly possible that this intensity factor is the primary source of uncertainty, as in the Stein (2009) model. Our specification facilitates inference on the source.

4.1 Momentum behaviour, crowding, and returns

We use quarterly holdings from the Thomson Reuters Institutional (13F) database starting at the end of the first quarter of 1980 until the end of the third quarter of 2015, and stock data from CRSP in the same period. All prices and shares held by institutions are adjusted with the CRSP adjustment factors. To construct momentum trading variables we use indicator variables for the standard winner and loser portfolios defined using NYSE decile breakpoints. We consider only common stocks (CRSP share code 10 and 11) listed on AMEX, NYSE and Nasdaq, and construct our measures using flow-adjusted net purchases of momentum stocks.

Daily and monthly momentum returns are obtained from Kenneth French's online data library, for March 1980 through December 2015. The momentum return at time t is defined as the return of the winners (those in the top 10% of the distribution according to returns from months $t-12$ to $t-2$) minus the return of the losers (those in the bottom 10% of the same distribution).[4] The decile cut-off points are determined using only NYSE listed stocks to avoid undue influence of micro-cap stocks. The returns of the winners and the losers portfolios are value-weighted within each decile.

At the end of quarter t, 13F institution i ($i = 1, \ldots, N_t$) has capital under management

$$K_{i,t} \equiv \sum_{j=1}^{J} P_{j,t} w_{i,j,t},$$

where $j = 1, \ldots, J$ indexes stocks and $w_{i,j,t}$ is the adjusted number of shares held by institution i in stock j, and $P_{j,t}$ is the adjusted price of stock j. The net flow of institution i is

$$Flow_{i,t} = K_{i,t} - K_{i,t-1}(1 + r_{i,p,t}),$$

where $r_{i,p,t} \equiv \sum_{j=1}^{J} \frac{P_{j,t-1} w_{i,j,t-1}}{K_{j,t-1}} r_{j,t}$ is the return on the portfolio held at the end of quarter $t-1$ and $r_{j,t}$ is the return on stock j during quarter t. We allocate flows to the beginning of period portfolio to measure buying net of flow-induced trading. The expected number of shares held at time t given

[4]The last month is skipped to avoid confounding with the short-term reversal effect of Jegadeesh [1990].

flow is $1 + Flow_{i,t}/K_{i,t-1} = K_{i,t}/K_{i,t-1} - r_{i,p,t}$), and the flow-adjusted net purchase of momentum stocks is

$$Buy_{i,t} = \sum_{j=1}^{J} P_{j,t}\left(w_{i,j,t} - w_{i,j,t-1}\left(\frac{K_{i,t}}{K_{i,t-1}} - r_{i,p,t}\right)\right)\iota_{j,t}, \tag{23}$$

where $\iota_{j,t}$ is an indicator variable that takes the value 1 if the stock is in the winner decile on the portfolio formation date t, -1 if it is in the loser in the decile, and zero otherwise. $Buy_{i,t}$ measures trading up to the rank date. We also consider a measure of slower-moving momentum capital $BuyP1_{i,t}$ which measures buying 'plus one' quarter after the rank date:

$$BuyP1_{i,t} = \sum_{j=1}^{J} P_{j,t+1}\left(w_{i,j,t+1} - w_{i,j,t}\left(\frac{K_{i,t+1}}{K_{i,t}} - r_{i,p,t+1}\right)\right)\iota_{j,t}. \tag{24}$$

The variables $Buy_{i,t}$ and $BuyP1_{i,t}$ do not directly proxy momentum capital. They are preliminary variables that we use to categorize institutions as momentum traders with the indicator function $\mathbb{1}_{[\cdot]}$. For example, $\mathbb{1}_{Buy_{i,t}} = 1$ if $Buy_{i,t} > 0$ and zero otherwise.

The third preliminary variable used in constructing our crowding measures is a measure of capital intensity

$$Cap_{i,t} = \sum_{j=1}^{J} P_{j,t} w_{i,j,t} \iota_{j,t}.$$

$Cap_{i,t}$ is meant to reflect the optimal trading of a representative momentum investor as set up in the model. Because all other momentum investors likewise follow this representative demand intensity, $Cap_{i,t}$ does not represent a source of randomness in the model; crowding uncertainty stems from the number of peer institutions that adopt a momentum strategy, $\mathbb{1}_{[\cdot]}$. Of course, this sourcing of crowding uncertainty need not be the case; either count or intensity could drive ambiguity in investors' anticipation of concurrent crowding. Our proxies are designed to shed light on this empirical matter.

We construct two sets of proxies based on the construction of $\mathbb{1}_{[\cdot]}$, generically referenced as

_1qtr and _4qtr measures. The _1qtr set are defined as

$$\text{Cnt\_1qrt}_t = \frac{1}{N_t}\sum_{i=1}^{N_t} \mathbb{1}_{Buy_{i,t}}, \tag{25}$$

$$\text{CntP1\_1qrt}_t = \frac{1}{N_t}\sum_{i=1}^{N_t} \mathbb{1}_{BuyP1_{i,t}}, \tag{26}$$

$$\text{Cap\_1qrt}_t = \frac{\sum_{i=1}^{N_t} Cap_{i,t}\mathbb{1}_{Buy_{i,t}}}{\sum_{i=1}^{N_t} K_{i,t}}, \tag{27}$$

these measures classify institutions as momentum investors based only on trading in the most recent quarter. The _4qtr set of measures are defined similarly

$$\text{Cnt\_4qrt}_t = \frac{1}{N_t}\sum_{i=1}^{N_t} \mathbb{1}_{\sum_{l=0}^{3}\mathbb{1}_{Buy_{i,t-l}}=4}, \tag{28}$$

$$\text{CntP1\_4qrt}_t = \frac{1}{N_t}\sum_{i=1}^{N_t} \mathbb{1}_{\sum_{l=0}^{3}\mathbb{1}_{BuyP1_{i,t-l}}=4}, \tag{29}$$

$$\text{Cap\_4qrt}_t = \frac{\sum_{i=1}^{N_t} Cap_{i,t}\mathbb{1}_{\sum_{l=0}^{3}\mathbb{1}_{Buy_{i,t-l}}=4}}{\sum_{i=1}^{N_t} K_{i,t}}, \tag{30}$$

except that they employ a more stable and persistent (and less timely) classification of institutions, requiring consistent momentum trading for four consecutive quarters. Note that both the _1qtr and the _4qtr versions of the Cap_ measure observe trade intensity only in the most recent quarter; i.e., the measures differ only with respect to the classification of institutions.[5] In short, Cnt_ measures seek to identify the fraction of 13F institutions supplying momentum capital to the market, whereas Cap_ measures add an overlay to identify the representative intensity with which that capital is applied. In the model the former is the source of ambiguity in crowding; the latter is a deterministic function of the setting.

In addition to these various measures for crowding we consider three specifications, as per the model. Thus, $\text{Crowd}_{\text{q-1}}$ is a generic reference to the level of the measure, and is our proxy for anticipated crowding. $\Delta\text{Crowd}_{\text{q}}$ is a generic reference to the change in the measure, and is our proxy for unanticipated crowding. $\hat{\sigma}_{\text{Crowd}}$ is a generic reference to the expected volatility of the measure using a GARCH(1, 1) specification, and is our proxy for crowding uncertainty. $\Delta\text{Crowd}_{\text{q}}$

[5]On occasion we generically refer to these as Cnt_ and Cap_ measures.

is generally our primary variable of interest, as it proxies the key dimension of crowding as goes the moments of momentum factor returns.

Table 1 provides summary statistics for the 13F data (Panel A); proxies for momentum investing (Panel B); and returns of the strategy (Panel C). In Panel A, an institution is considered a momentum investor if they are classified as such by one of our measures for at least 2/3 of the available quarters. By this determination only 22% (1414/6360) of institutions consistently follow a momentum strategy. By contrast, Grinblatt et al. [1995] find that 77% of mutual funds are momentum investors. The difference is likely attributable to a difference in definition[6] and the fact that 13F data is at the institution, rather than portfolio, level.

[Insert Table 1 near here]

Also from Table 1, Panel A, momentum institutions differ materially from other institutions. Momentum institutions have a higher turnover (24% compared to 21%), manage more assets (2.46 billion versus 1.23 billion), and hold a more diversified collection of portfolios (213 stocks on average versus 123).

Table 1, Panel B provides descriptive statistics of the crowding variables. Using the mean of either Cnt_ measure with the $\text{Crowd}_{q\text{-}1}$ specification at a 1qtr horizon, we infer that approximately 50% of institutional investors are classified as momentum investors in a given quarter. This suggests the importance of identifying *consistent* momentum investors rather than just aggregating institutional trading in momentum stocks—in a given quarter the aggregate is roughly in balance. Nevertheless, the identity of institutions trading with momentum is persistent, as can be seen with the 4qtr measures. These average 11.7% for Cnt_ and 10.2% for CntP1_, compared with the $0.5^4 = 6.25\%$ value implied under a null hypothesis of no momentum-trading institutions (each trades randomly with and against momentum in a given quarter). Also from Panel B, most crowding variables show strong persistence as captured by their coefficients in an AR(1) regression. Given this evidence of persistence, we estimate the volatility of the crowding measures using the residuals from the respective AR(1) regressions. We adopt the usual GARCH(1,1) specification of Bollerslev [1986], to estimate the conditional volatility of the residuals.

[6]Grinblatt et al. [1995] define momentum investors each quarter if they buy the winners and sell the losers as defined by the returns over that same quarter.

Table 1, Panel C provides the output of a regression of momentum returns using two risk models: the Fama-French 3 factor model (abbreviated FF3) and a dynamic version of the same model (DFF3). We consider these specifications, as well as raw momentum returns, in parallel throughout our analyses.[7] Momentum has a negative exposure to the market, size, and value factors, hence risk-adjusted momentum returns are even larger than raw returns (monthly alpha 1.4 - 1.6%, as in Asness et al. [2013] and Asness et al. [2014]).

Grundy and Martin [2001] show that the momentum portfolio has strongly time-varying risk exposures due to its rapidly changing composition. In the DFF3 specification we include regressors with an interaction dummy variable that takes the value 1 if the factor has a positive return in the previous year and zero otherwise (D preface in the variable names). Consistent with Grundy and Martin [2001], we find that the betas of momentum respond significantly to lagged returns on the market (t-stat of 2.8) and the value factor (t-stat of 2.1). The dynamic model better captures the three-factor risks of momentum, as the R-squared of the regression increases from 12% to 25%. However, as pointed out in Barroso [2014], time-varying risk exposure does not explain the alpha of the strategy.

In unreported results we also find that momentum has substantial crash risk in our sample, combining high excess kurtosis with pronounced left-skewness. This is unsurprising as our sample period includes the eventful momentum crash of March-May 2009.

Table 2 Panel A considers the persistence of our momentum measures using the transition probability for the classification of an individual institution, e.g., $\mathbb{1}_{Buy_{i,t}}$ and $\mathbb{1}_{\sum_{l=0}^{3}\mathbb{1}_{BuyP1_{i,t-l}}=4}$. For all measures the probability of maintaining the current momentum classification in the following quarter is more than 50%, and it is 69% (64%) according to _4qtr measures. This implies persistence. Transition probabilities for momentum versus non-momentum institutions are more comparable at a four-quarter horizon when using _4qtr measures. Using the case of Cnt_, the probability of transitioning to a momentum-trading institution four quarters ahead is 29% for an institution currently classified as momentum, versus an unconditional probability of 11% and a probability of 9% for a current non-momentum trader. The results in the case of CntP1_ are similar. Thus, the _4qtr measures provide a meaningful identification of momentum-trading institutions.

[7] We often make reference to FF3 or DFF3 models or residuals. In fact, in the case of returns regressions the dependent variable is the raw momentum return and the FF3 or DFF3 regressors are included as controls, and for the crash and volatility analysis the residuals from the FF3 or DFF3 models are used.

[Insert table 2 near here]

One possible concern is that investors' preference for certain sectors or investment styles make them trade persistently in the direction of (or against) momentum for several quarters in a row. This possibility is challenged by the rapidly changing composition of the momentum portfolio itself. To illustrate, Table 2 Panel B shows the persistence of stocks' membership in either leg of the momentum portfolio. Winners have a 55% chance of remaining winners the following quarter, but at four quarters the likelihood is only 16%, which is actually less than the 23% chance of becoming a loser. Persistence is higher with losers, with 64% of loser stocks retaining that classification the following quarter and 31% retaining it after four quarters.

All in all, Tables 1 and 2 suggest that our proxies for crowding – particular _4qtr measures – exhibit a level of persistence at the individual institution level that suggests successful identification of institutions maintaining a purposeful allocation of capital towards an ever-changing momentum portfolio. The question then is whether these proxies can be used to identify the crowding effects detailed in the model.

4.2 Crowding and conditional expected returns on the momentum factor

Table 3 shows the results of predictive regressions of momentum returns on the various crowding measures. All momentum trading measures are appropriately lagged (in this and subsequent tables) to ensure that there is no overlap between the measurement of the independent variable and the momentum return. For example, we use the change in Cnt_1qrt_t to predict momentum returns in the quarter $t + 1$, and the change in CntP1_1qrt_t to predict momentum returns in $t + 2$. As a control we include the lagged realized volatility of momentum computed from the squared daily returns of the WML (winner minus loser) portfolio in the previous quarter. Barroso and Santa-Clara [2015] show that this strongly predicts (negatively) momentum returns.[8] We consider three specifications for the dependent variable – the return on the momentum factor (raw), and its risk-adjusted returns using both the static (FF3) and dynamic (DFF3) Fama-French 3 factor models. Finally, because computing the regressors requires up to six quarters of data, the regression sample begins in September 1981 and ends in December 2015.

[8]In unreported results we also controlled for the bear market states proposed by Cooper et al. [2004]. Using this control did not change our results.

[Insert table 3 near here]

The model predicts that both anticipated crowding (proxied with $\text{Crowd}_{\text{q-1}}$) and unanticipated crowding (proxied with $\Delta\text{Crowd}_{\text{q}}$) negatively relate to momentum returns, as momentum capital pushes rank-date valuations closer to fundamental value. However, the effect for unanticipated crowding should be larger for three reasons: (1) it cannot be countered ex ante with an optimal demand response, (2) it induces a feedback effect as pointed out in Stein (2009), and (3) it potentially triggers a reversal of positions as momentum investors subsequently learn that their collective investment in the strategy is too high. We see statistically significant evidence of this at the 1% level[9] in Table 3 with each of the three specifications of the dependent variable—with two categorical exceptions. First, the statement applies in Panel A with the _4qtr identification measures, but not in Panel B where statistical reliability is lost with the noisier _1qtr identification measures. Second, the statement applies to Cnt_ and CntP1_ measures but not the Cap_ measure. This difference is consistent with the model set up in which momentum investors find it relatively difficult to condition on the number of peers engaging in the strategy, but relatively easy to condition on how intensely each will choose to trade under current circumstances.

The regressions in Table 3 do not present an unambiguous relation for the levels specification, $\text{Crowd}_{\text{q-1}}$. While the estimates are generally consistent with a negative relation, in the case of Cap_ the estimate is significantly positive. If $\text{Crowd}_{\text{q-1}}$ indeed relates to the anticipated level of momentum capital, and Cap_ indeed relates to the anticipated intensity of deployment, then this result potentially fits the model. To wit, anticipated capital does not generate feedback, attenuating the negative influence of demands. If that attenuation is sufficient, it is possible that the positive relation between returns and the optimal response of the representative demand curve generates the positive coefficient on Cap_. This may be pushing the interpretation of the proxies beyond their capacities. Nevertheless, this intriguing possibility is further confirmed when we estimate the coefficients on Cnt_ and Cap_ measures jointly in Table 7.

Finally, from Table 3 the anticipated volatility of crowding from the GARCH (1, 1) forecast, $\hat{\sigma}_{\text{Crowd}}$, is positively related to the expected return of momentum at the 1% level for one specification (CntP1_ and DFF3 returns), but the relation is generally insignificantly positive. A positive

[9]Unless otherwise noted the significance levels discussed refer to two-tailed tests even when the model provides a clear prediction for the sign of the coefficient.

relation potentially indicates that uncertainty about the number of competing momentum investors inhibits participation in the strategy, but that inference is at best cloudy. In our later consideration of volatility in momentum returns (Table 5) we find evidence consistent with this story, but again the evidence is cloudy.

4.3 Crowding and negative tail events in momentum returns

Of particular interest is the relation between unanticipated crowding in momentum and the pronounced left tail in the distribution of momentum returns. This left tail is well-known empirically, and shown in the model to be an implication of the perverse feedback effect of crowding identified by Stein (2009). To assess this link between the left tail of momentum returns and crowding, we run probit regressions using indicators for a return below the 10^{th} and below the 20^{th} percentile of the distribution, controlling for lagged volatility of momentum returns. This control is important, as volatility predicts mass in both tails, whereas unanticipated crowding predicts mass in the left tail.

[Insert Table 4 near here]

Table 4 presents the results. We focus on Panel A with the 10% tails; results in Panel B for the 20% tails are consistent. To save space, Table 4 only considers the _4qtr measures and the raw and DFF3 return specifications. Consistent with the argument that unanticipated peers are the primary source of crowding, we find a statistically reliably positive relation between $\Delta\text{Crowd}_{\text{q}}$ and subsequent left-tail events using Cnt_ and CntP1_ measures. The relation between the change in Cap_ measures and left-tail events is also positive, but not statistically reliable. This weaker link is again consistent with the conjecture that capital intensity is endogenous to the optimized demands of momentum traders, who each follow a deterministic demand curve; whereas the number of peers competing for the trade is exogenous to the traders' calculations.

Neither the level ($\text{Crowd}_{\text{q-1}}$) nor the volatility of crowding ($\hat{\sigma}_{\text{Crowd}}$) reliably predicts left-tail events. This is to be expected if $\text{Crowd}_{\text{q-1}}$ indeed proxies for anticipated crowding, as anticipation prevents a skewed impact on momentum returns. Likewise, prediction regarding the volatility of crowding is not nearly so sharp as predictions regarding the direct measure of unanticipated crowding, $\Delta\text{Crowd}_{\text{q}}$.

That ΔCrowd_q predicts a fat left tail in momentum returns is a powerful support for Stein's (2009) argument regarding the perverse effects of unanticipated crowding. While Stein's argument is general to unanchored strategies, our tests of a related model specifically developed for momentum suggests that the result obtains more generally. Crowding seems to be a first order consideration in asset pricing.

4.4 Crowding and the volatility of momentum returns

In this section we present predictive regressions for realized volatility of momentum returns, computed from raw and risk adjusted (both static and dynamic) daily returns over the quarter. In all cases we include lag realized volatility as a control.

[Insert Table 5 near here]

Table 5 presents the results. First, note that lagged realized volatility has very strong predictive power for subsequent volatility, with t-statistics between 6.7 and 8.5 across all regressions. This confirms the results of Barroso and Santa-Clara [2015] who show that momentum has highly predictable volatility. Table 5 also shows that uncertainty about crowding, as proxied with $\hat{\sigma}_{\text{Crowd}}$, also predicts positively the subsequent volatility of momentum returns. The coefficients have the predicted positive sign for all proxies and 7 out of 18 show significance at the 5% level, with a couple more at the 10% level. This suggests that crowding is an important determinant of risk in momentum returns.

The observation that $\hat{\sigma}_{\text{Crowd}}$ predicts positively risk in the momentum strategy supports the earlier finding in Table 3 suggesting (weakly) that $\hat{\sigma}_{\text{Crowd}}$ predicts positively momentum returns. If momentum investors pull back from the strategy when $\hat{\sigma}_{\text{Crowd}}$ is high in the belief that $\hat{\sigma}_{\text{Crowd}}$ predicts risk, a positive relation in Table 3 follows. The evidence in Table 5 supports that belief. One final observation is that the forecasting power of $\hat{\sigma}_{\text{Crowd}}$ in Table 5 is strongest with the _1qtr measures. Presumably stability and persistence in the _4qtr measures – an advantage in the ΔCrowd_q and Crowd_{q-1} specifications – is a disadvantage when it comes to proxying uncertainty.

Table 5 provides a fairly robust inference that crowding predicts negatively volatility in momentum returns. The coefficient estimate on both the ΔCrowd_q and the Crowd_{q-1} specifications (applied to Cnt_ measures) is statistically significant at the 5% level in 13 of 24 cases across Panels A and B, and is negative in all cases. One possible explanation for this is a reversal of causality, with expectations of risk in the momentum strategy affecting institutions' willingness to participate in the strategy (i.e., influencing the Cnt_ measures). This would require that forward-looking institutions observe a wider information set than just lagged volatility in momentum returns, but that is surely plausible. We present evidence relating to this conjecture in the next section.

Finally, note that the explanatory power of crowding measures pales in comparison to that of lagged realized volatility. This is perhaps not surprising as lag dependent variables capture all persistent characteristics of the setting. Moreover, it is estimated at a daily frequency whereas crowding measures are based on holdings data observed at a quarterly frequency. What the crowding measures have going for them is that they are explicit economic measures brought to bear on the data from theory. Lag dependent variables offer little insight beyond persistence in setting. We believe that this fact makes up for the shortcoming in predictive power.

4.5 Determinants of crowding

Barroso and Santa-Clara [2015] show that volatility strongly predicts negatively the performance of the momentum strategy. As such, rational momentum investors should reduce their exposure to the strategy whenever recent volatility is high. The previous section alludes to the possibility that this concern for risk indeed attenuates crowding. Here we explore the matter directly using lagged realized volatility as a predictor of crowding.

We also consider the lagged first moment of momentum returns, motivated by several studies. Chabot et al. [2014] show in a comprehensive sample period of 140 years that the crash risk of momentum increases after periods of good recent returns in the strategy. If such periods also relate to crowding, particularly unanticipated crowding, the model implications may be involved. Piazzesi and Schneider [2009] also use survey data to study the presence of momentum investors in the US housing market. They find there is evidence of a time-varying subset of momentum investors that doubles in size towards the end of the boom in the housing market. We examine the matter in equities.

Table 6 shows the results of regressions of crowding on one-year returns and one-year volatility for the momentum factor, computed using daily observations and lagged at the indicated horizon. Because 13F observations of institutional trading occur over a quarter, we lag the returns and volatility by a quarter. Also, to ensure predetermined values for the _4qtr measures, we add one-year returns and volatility lagged five quarters. We use the level of Cnt_ and Cap_ measures as dependent variables.

[Insert Table 6 near here]

We find that one-year returns predict positively crowding in momentum. The coefficients on lagged returns from Table 6 are positive in all six regressions and statistically significant at the 1% level in eight. In the case of overlap between the estimation period for _4qtr measures and returns (i.e., top row), the relation is less positive due to the negative effect of crowding on returns. However, in the case of returns lagged five quarters, there is a reliable positive relation with Cnt_ measures, but not Cap_ measure. Results are more comparable at different horizons in the case of _1qtr measures. The evidence here confirms that lagged returns positively influence crowding.

A noteworthy pattern in the relation between lagged returns and crowding is the shift in strength at various horizons: Cnt_ measures react stronger at the five-quarter lag whereas Cap_ reacts strongest at the one quarter lag. This effect is not attributable to overlap concerns, as it is seen with _1qtr measures as well. This suggests that the intensity of the representative demand curve of existing momentum investors' responds relatively quickly to past returns, but that institutions' adoption and termination of the strategy occurs with a more delayed response. This makes sense as the choice of investment strategy is surely a weightier decision than the parameterization of an existing strategy.

Regarding one-year volatility, from Table 6 we also find predictability for the crowding variables. In the case of _4qtr measures the dependence of crowding on momentum-return volatility occurs only when the observation periods overlap, obfuscating the sequencing of events. Indeed, from Table 5 we have already seen that Cnt_ measures of crowding predict negatively future risk in the momentum strategy. From Table 6, crowding also *reacts* negatively to past risk in the momentum strategy, but only at a relatively high frequency (one quarter). This is seen most clearly in the case of the _1qtr measures where there is no overlap and there is a statistically reliable (at the 1% level) reduction in crowding. Combining inferences from the anticipation and reaction analyses (i.e., Tables 5 and 6), we conclude that risk is a primary determinant of crowding; that its impact is largely anticipatory; and that the response is fast—materially faster than the reaction to past momentum returns.

Interestingly, this observation applies to entry and exit to the strategy (Cnt_ measures) rather than to the intensity of trading, where the relation is not statistically reliable. Note that if an institution reverses its momentum holdings, the input to the indicator function $\mathbb{1}_{Buy_{i,t}}$ turns negative; the indicator turns to zero; and Cnt_ measures fall. The evidence in Table 6 suggests, therefore, that risk in the strategy doesn't merely reduce the intensity of momentum demands (as proxied with Cap_ measures), it induces wholesale exit from the strategy. Barroso and Santa-Clara [2015] suggest that momentum investors should scale their position according to recent volatility.[10] These results suggest that institutions, in aggregate, react in a rather more dramatic fashion.

Cooper et al. [2004] show that momentum returns are stronger in bull markets. That evidence supports the interpretation of momentum as partially caused by over-confident and self-attributing investors becoming particularly over-confident during bull markets [Daniel et al., 1998]. In unreported results we do not find any predictive power of market states for our measures of momentum crowding once controlling for the lagged returns and volatility of the strategy.

Our results are consistent with the momentum strategy becoming more crowded when its recent performance is good both in terms of high returns and low volatility. The volatility results suggest that forward-looking rational momentum investors successfully time risk in the strategy. That may be partly causal as exodus from the strategy potentially generates self-fulfilling risk. On the other hand, chasing momentum returns is harder to rationalize since returns to the strategy do not show time-series autocorrelation, indeed recent high returns seem to increase the crash risk for the strategy [Chabot et al., 2014]. However, if the response to past returns is more euphoric than rational, our evidence on the role of crowding in predicting subsequent momentum performance offers a possible explanation for the Chabot et al. result.

[10]They examine the benefits of risk management with different windows to compute volatility, all shorter than one year.

4.6 Capital versus crowd

We have argued that Cnt_ measures track the number of momentum investors and Cap_ measures track the intensity of their trade. In our final analysis we consider the two dimensions of crowding in a multivariate setting. While the results largely confirm the preceding analyses, this analysis serves to highlight the differing nature of the two sets of proxies. As the intent is to summarize, we consider all three moments of momentum collectively in a single table: returns; left-tail events; and volatility. We only present one case, that of residual returns from the dynamic Fama-French model using the Cnt_ (rather than CntP1_) identification of the number of momentum-trading institutions.

[Insert Table 7 near here]

Results for returns are presented in the first two columns of Table 7, for the _1qtr and _4qtr measure, respectively. In comparing the columns, it is clear that the more stable and persistent _4qtr measure does a better job of predicting momentum returns. This highlights the importance of identifying momentum-trading institutions, rather than just aggregating institutional trading in momentum stocks, in studying the effects of crowding. From the _4qtr column it is also clear that the *number* of momentum investors, rather than the *intensity* of their trading, is most relevant to the crowding story developed in Stein (2009). This is further supported by the strength of the relation for $\Delta \text{Crowd}_{\text{q}}$, which we argue proxies for unanticipated crowding. Note also that the positive relation of $\hat{\sigma}_{\text{Crowd}}$ in predicting momentum returns is here statistically significant at the 1% level.

In contrast with the negative predictive relation of Cnt_ measures for returns, the Cap_ measure is significantly positively related to future returns using the $\text{Crowd}_{\text{q-1}}$ specification. Because the Table 7 regressions estimate marginal effects and the marginal difference in the Cap_ measure is the intensity of the representative momentum trade, this evidence suggests that the important dimension of a feedbacking effect from crowding is the number of institutions, rather than the manner in which they trade. The positive coefficient estimate supports the view that $\text{Crowd}_{\text{q-1}}$, using the Cap_4qtr measure, captures optimal intensity in the momentum trade. According to the model, that optimum is high when (or rather anticipating that) the momentum on the stocks in question is also high.

The left-tail results in Table 7 confirm the results in Table 4: unanticipated crowding contributes to a fat negative tail in the distribution of momentum returns, and the effect is attributable to uncertainty in the number of momentum-trading institutions. Volatility results likewise confirm the earlier analysis (Table 5).

5 Conclusion

We provide a model based on supply and demand considerations for the momentum factor to study how momentum moments arise due to the trading behavior of investors. Our model delivers intuitive predictions such as that momentum factor returns are decreasing in changes in momentum capital and increasing in the uncertainty about momentum capital, but also novel predictions regarding momentum's second and higher moments. We obtain that uncertainty about momentum capital predicts momentum return volatility and that large unanticipated changes in momentum capital may lead to crashes in momentum strategy returns.

Using quarterly holdings of 13F institutions in the period from 1980 to 2015, we construct several proxies for momentum capital based on aggregate momentum trading to test the predictions of our model. We find that changes in our proxies negatively predict momentum strategy returns. We also find evidence that uncertainty about momentum capital predicts higher momentum volatility, and that our measures of momentum trading are positively related to momentum crashes. Our empirical findings are generally in line with the idea that momentum trading and the uncertainty thereof contribute to momentum's moments.

A Derivations

A.1 Derivation of Eq. (8)

First notice that solving Eq. (7) is equivalent to solving each of the following

$$\max \quad \frac{K_0^{1-\rho}}{1-\rho} \cdot E\left[e^{(1-\rho)g}\right] \quad \Leftrightarrow \quad \min \quad \log E\left[e^{(1-\rho)g}\right],$$

(presuming $\rho > 1$). Second, to solve for the fraction of wealth invested in the risky portfolio, we follow Campbell and Viceira [2002, appendix] and approximate the objective function using a second-order Taylor expansion of g around $r_p - f = 0$:

$$\begin{aligned} g &\cong f + \log(1) + \frac{\varsigma e^0}{1+\varsigma(e^0-1)}\left(r_p - f\right) + \frac{1}{2}\frac{\varsigma\left[e^0\left(1+\varsigma\left(e^0-1\right)\right) - \varsigma e^{2\cdot 0}\right]}{\left(1+\varsigma(e^0-1)\right)^2}\left(r_p - f\right)^2, \\ &\cong f + \varsigma\left(r_p - f\right) + \frac{1}{2}\left(\varsigma - \varsigma^2\right)\sigma^2, \end{aligned} \tag{31}$$

where $\left(r_p - f\right)^2$ is replaced with its expected value σ^2. Using (31), we can then rewrite the maximization problem to

$$\begin{aligned} &\min \quad \left(\{(1-\rho) f\} + \log E\left[\exp\left[\frac{1}{2}\left(\varsigma - \varsigma^2\right)(1-\rho)\sigma_P^2\right] \cdot \exp\left[\varsigma(1-\rho)\left(r_p - f\right)\right]\right]\right), \\ \Leftrightarrow &\min \quad \left(\frac{1}{2}\left(\varsigma - \varsigma^2\right)(1-\rho)\sigma_P^2 + \varsigma(1-\rho)(\mu_P - f) + \frac{1}{2}\varsigma^2(1-\rho)^2\sigma_P^2\right), \\ \Leftrightarrow &\max \quad \left(\varsigma\left(\mu - f + \frac{1}{2}\sigma^2\right) - \frac{1}{2}\varsigma^2\rho\sigma^2\right), \end{aligned}$$

which has solution

$$\varsigma = \frac{\mu - f}{\rho\sigma^2}. \tag{32}$$

We now determine μ and σ^2 in the context of a portfolio comprised of the market investment plus a long-short momentum investment. Because the momentum portfolio is self-financing, feasible combinations of the market portfolio and the momentum portfolio are given by the weight vector $\boldsymbol{w}' = \begin{bmatrix} 1 & w_m \end{bmatrix}$, i.e., hold the market portfolio plus a proportionate long-short momentum overlay w_m. The optimal risky portfolio is then that choice of w_m that solves the constrained optimization

$$\min_{\boldsymbol{w}} \qquad \frac{\boldsymbol{w}'\boldsymbol{\Sigma}\boldsymbol{w}}{2}, \qquad \text{s.t.} \qquad \boldsymbol{\mu}'\boldsymbol{w} = r^* - f.$$

using weights $\boldsymbol{w}' = \begin{bmatrix} 1 & w_m \end{bmatrix}$, where

$$\boldsymbol{\mu} = \begin{bmatrix} r - f \\ Em_{Rnk\to 1} \end{bmatrix}, \qquad \boldsymbol{\Sigma} = \begin{bmatrix} \sigma_\chi^2 & 0 \\ 0 & \sigma_\delta^2 \end{bmatrix},$$

and $r^* - f$ is a target return premium that traces out the efficient frontier (recall r is the required return on the market portfolio). This has solution

$$w_m = \frac{Em_{Rnk\to 1} \Big/ \sigma_\delta^2}{(r-f) \Big/ \sigma_\chi^2}. \tag{33}$$

Using (33), the parameters of the optimal risky portfolio are

$$\mu_p - f = \begin{bmatrix} r - f & E\left[m_{Rnk\to 1}\right] \end{bmatrix} \begin{bmatrix} 1 \\ w_m \end{bmatrix} = r - f + w_m E\left[m_{Rnk\to 1}\right], \tag{34}$$

and

$$\sigma_p^2 = \boldsymbol{w}'\boldsymbol{\Sigma}\boldsymbol{w} = \sigma_\chi^2 + w_m^2 \sigma_\delta^2 = \frac{\sigma_\chi^2}{r-f}\left(r - f + w_m E\left[m_{Rnk\to 1}\right]\right). \tag{35}$$

Taking the ratio gives

$$\varsigma = \frac{r-f}{\rho \sigma_\chi^2}. \tag{36}$$

Combining (33) and (36),

$$Demand = w_m \cdot \varsigma \cdot K_0 = \frac{Em_{Rnk\to 1}}{\rho \sigma_\delta^2} K_0. \tag{37}$$

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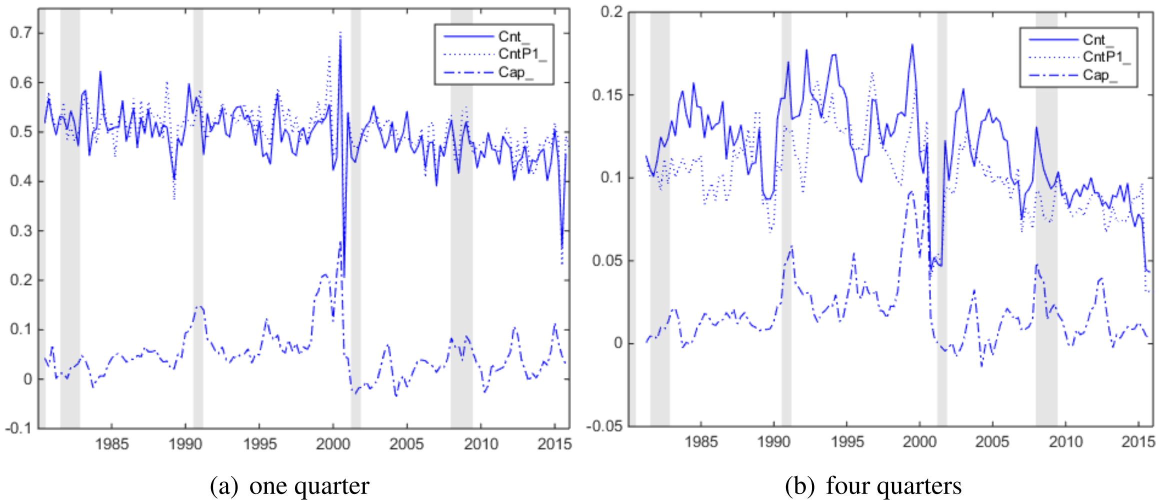

Figure 1: Measures of crowding

Panel (a) and (b) report the _1qrt and _4qrt crowding measures, respectively, constructed with 13F holdings data in the period from 03/1980 to 09/2015.

Table 1: Descriptive statistics of momentum returns

In Panels A and B the indicated variable is computed by institution (i.e., 13F filer) and then summarized across institutions. Qtrs, med., stdev., and mgd. refer to quarters, median, standard deviation, and managed, respectively. Assets are in units of $100 million and turnover is quarterly. Momentum investors refer to institutions classified as a momentum trader by one of our measures for at least 2/3 of the available quarters. Crowd refers to Cnt_1qrt, Cnt_1qrtP1, and Cap_1qrt; likewise the _4qrt extensions (as defined in Section 5.1). $\hat{\sigma}_{\text{Crowd}}$ is the estimate of volatility from a GARCH(1,1) model. Panel C contains factor exposures of quarterly momentum returns on the Fama-French three factor model and a dynamic extension with the three factors interacted with their past annual returns. Alphas are monthly and t-statistics use White standard errors.

Panel A. Institutions

	all			momentum investors			not momentum investors		
	mean	med.	stdev.	mean	med.	stdev.	mean	med.	stdev.
#Qtrs of data	34.3	24.0	32.1	34.3	24.0	32.1	34.3	24.0	32.1
#Qtrs missing	3.6	0.0	9.5	3.6	0.0	9.5	3.6	0.0	9.5
#Stocks held	143.2	62.9	275.5	213	90.1	372.5	123.1	55.2	236.6
Assets mgd.	15.2	2.0	102.4	24.6	2.1	161.6	12.3	1.9	76.6
Turnover	0.21	0.16	0.17	0.24	0.18	0.18	0.21	0.15	0.16
#Institutions	6360			1414			5059		

Panel B. Crowding variables

	Cnt_				CntP1_				Cap_			
	mean	med.	stdev.	ar(1)	mean	med.	stdev.	ar(1)	mean	med.	stdev.	ar(1)
_1qtr measure:												
ΔCrowd	0.000	0.004	0.075	-0.579	0.000	0.002	0.077	-0.563	0.000	0.001	0.034	-0.092
Crowd	0.492	0.495	0.055	0.083	0.504	0.509	0.054	0.001	0.050	0.042	0.051	0.782
$\hat{\sigma}_{\text{Crowd}}$	0.052	0.049	0.010	0.367	0.053	0.052	0.005	0.523	0.029	0.023	0.018	0.703
_4qtr measure:												
ΔCrowd	-0.001	0.000	0.017	-0.153	-0.001	0.000	0.017	-0.181	0.000	0.001	0.012	0.029
Crowd	0.117	0.120	0.029	0.818	0.102	0.101	0.026	0.770	0.019	0.014	0.020	0.810
$\hat{\sigma}_{\text{Crowd}}$	0.017	0.016	0.003	0.881	0.018	0.017	0.003	0.756	0.011	0.008	0.007	0.666

Panel C. Returns

	FF3					dynamic FF3							
	alpha	mkt	SMB	HML	R^2	alpha	mkt	SMB	HML	Dmkt	DSMB	DHML	R^2
coefficient	0.016	-0.35	-0.48	-0.59	12%	0.014	-0.85	-0.68	-0.95	0.81	0.48	1.02	25%
t-statistic	(4.4)	(-2.0)	(-2.1)	(-2.1)		(4.3)	(-2.9)	(-2.7)	(-2.7)	(2.8)	(1.2)	(2.1)	

Table 2: Transition frequencies

The table presents the probability of transitioning from the event in the row heading to that in the column heading at the indicated time (t indexes quarters), conditional on the later period not containing a missing observation. Panel A tabulates the transition at the level of individual institutions in terms of the momentum classification used to construct the indicated crowding variable ('1' stands for being a momentum trader and '0' for not being a momentum trader). Panel B tabulates stocks' membership in the winner, loser, or middle deciles of the momentum ranking. 'All' refers to the unconditional probability of classification.

Panel A. Institutions' trading

		t+1		t+4		All
Indicator for > 0:		1	0	1	0	
Cnt_/Cap_1qrt	1	0.56	0.44	0.55	0.45	0.48
	0	0.41	0.59	0.41	0.59	0.52
Cnt_/Cap_4qrt	1	0.69	0.31	0.29	0.71	0.11
	0	0.04	0.96	0.09	0.91	0.89
Cnt_1qrtP1	1	0.55	0.45	0.54	0.46	0.50
	0	0.44	0.56	0.45	0.55	0.50
Cnt_4qrtP1	1	0.64	0.36	0.19	0.81	0.10
	0	0.04	0.96	0.08	0.92	0.90

Panel B. Stock returns

	t+1			t+4			All
	Win.	mid	Los.	Win.	mid	Los.	
Winner	0.55	0.42	0.02	0.16	0.60	0.23	0.13
mid	0.08	0.83	0.09	0.12	0.74	0.14	0.68
Loser	0.02	0.34	0.64	0.17	0.52	0.31	0.18

Table 3: Momentum factor returns on crowding measures

Each column represents a predictive regression of quarterly momentum factor returns (1981 - 2015) on crowding. Each panel presents three specification: (1) without controlling for risk-factors; (2) controlling for the Fama and French three factor model; and (3) controlling for the dynamic factor model with the three factors interacted with their past annual returns. Each set considers the three indicated crowding measures. The regressor 'Crowd' refers to the level of the crowding measure at the end of quarter q-1; $\Delta Crowd_q$ refers to the change over quarter q; and $\hat{\sigma}_{Crowd}$ is the estimate of volatility from a GARCH(1,1) model. 'Realized vol. of Mom rets.' is a control variable equal to the lagged realized volatility of daily momentum returns over the previous quarter (intercepts not tabulated). The t-statistics are computed with White standard errors.

Panel A. Crowding measures constructed using four-quarter trading histories

Model:	cumulative returns			FF3			dynamic FF3		
Measure:	Cnt_	CntP1_	Cap_	Cnt_	CntP1_	Cap_	Cnt_	CntP1_	Cap_
$\Delta Crowd_q$	-0.57	-0.59	-0.27	-0.57	-0.59	-0.32	-0.55	-0.62	-0.28
	(-2.8)	(-2.8)	(-1.0)	(-3.2)	(-3.6)	(-1.3)	(-2.8)	(-3.0)	(-0.9)
$Crowd_{q-1}$	-0.17	-0.04	0.40	-0.14	0.10	0.41	-0.11	0.09	0.48
	(-1.1)	(-0.3)	(1.8)	(-1.2)	(0.7)	(2.1)	(-1.0)	(0.7)	(2.5)
$\hat{\sigma}_{Crowd}$	2.39	2.76	-0.20	2.53	3.26	-0.04	1.81	2.42	-0.36
	(1.4)	(1.6)	(-0.3)	(1.7)	(2.4)	(-0.1)	(1.3)	(1.6)	(-0.6)
Realized vol.	-0.36	-0.32	-0.31	-0.36	-0.29	-0.32	-0.30	-0.23	-0.26
of Mom rets.	(-1.9)	(-1.8)	(-1.6)	(-2.6)	(-2.1)	(-2.4)	(-2.6)	(-1.9)	(-2.3)
Adj-rsquare	13.5%	14.4%	10.5%	26.6%	28.2%	24.4%	35.3%	37.6%	35.0%

Panel B. Crowding measures constructed using one-quarter trading histories

Model:	cumulative returns			FF3			dynamic FF3		
Measure:	Cnt_	CntP1_	Cap_	Cnt_	CntP1_	Cap_	Cnt_	CntP1_	Cap_
$\Delta Crowd_q$	-0.07	-0.06	-0.06	-0.07	-0.07	-0.11	-0.12	-0.09	-0.13
	(-1.0)	(-0.7)	(-0.7)	(-1.1)	(-0.9)	(-1.3)	(-1.3)	(-1.1)	(-1.2)
$Crowd_{q-1}$	-0.04	-0.04	0.14	-0.03	-0.01	0.12	0.02	0.00	0.17
	(-0.4)	(-0.4)	(1.4)	(-0.4)	(-0.1)	(1.5)	(0.2)	(-0.0)	(2.3)
$\hat{\sigma}_{Crowd}$	0.15	0.68	0.05	0.26	1.01	0.14	0.29	0.89	-0.02
	(0.5)	(0.9)	(0.2)	(1.0)	(1.7)	(0.7)	(0.7)	(1.1)	(-0.1)
Realized vol.	-0.29	-0.30	-0.32	-0.30	-0.32	-0.34	-0.23	-0.26	-0.28
of Mom rets.	(-1.6)	(-1.6)	(-1.6)	(-2.1)	(-2.1)	(-2.4)	(-1.8)	(-2.0)	(-2.4)
Adj-rsquare	6.9%	7.2%	9.9%	20.2%	21.0%	23.8%	33.8%	32.4%	35.9%

Table 4: Crowding and the left-tail of momentum returns

Each column represents a Probit regression with an indicator for next-quarter momentum returns in the bottom 10% of the full-sample (1981 - 2015) distribution (Panel A) or bottom 20% (Panel B). Each panel presents two sets of dependent variables: (1) the 3 month return on the momentum portfolio; and (2) its residual on the dynamic Fama and French model with the three factors interacted with their past annual returns. Each set considers the three indicated crowding measures. In all cases the crowing measure is constructed using a four-quarter trading history. The regressor 'Crowd' refers to the level of the crowding measure at the end of quarter q-1; $\Delta Crowd_q$ refers to the change over quarter q; and $\hat{\sigma}_{Crowd}$ is the estimate of volatility from a GARCH(1,1) model. 'Realized vol. of Mom rets.' is a control variable equal to the lagged realized volatility of daily momentum returns over the previous quarter (intercepts not tabulated).

	Panel A. Predicting the 10% left tail						Panel B. Predicting the 20% left tail					
Dependent variable:	cumulative returns			dynamic FF3 residuals			cumulative returns			dynamic FF3 residuals		
Crowding measure:	Cnt_	CntP1_	Cap_	Cnt_	CntP1_	Cap_	Cnt_	CntP1_	Cap_	Cnt_	CntP1_	Cap_
$\Delta Crowd_q$	26.94	28.92	15.56	22.41	44.92	9.66	25.06	11.79	-1.47	18.50	13.55	2.58
	(2.6)	(2.2)	(1.0)	(2.2)	(3.0)	(0.7)	(3.0)	(1.4)	(-0.1)	(2.3)	(1.5)	(0.2)
$Crowd_{q-1}$	12.15	10.17	-5.52	9.61	3.17	-7.78	10.52	5.62	-4.94	9.35	-0.35	-10.20
	(1.8)	(1.3)	(-0.6)	(1.5)	(0.4)	(-0.9)	(2.1)	(1.0)	(-0.7)	(1.9)	(-0.1)	(-1.4)
$\hat{\sigma}_{Crowd}$	30.71	48.36	22.31	52.30	10.47	22.66	1.26	-12.01	11.65	-10.26	-47.83	27.75
	(0.6)	(0.9)	(1.1)	(1.1)	(0.2)	(1.1)	(0.0)	(-0.2)	(0.6)	(-0.2)	(-0.9)	(1.4)
Realized vol.	13.70	11.82	10.81	13.70	12.58	11.94	8.61	7.39	6.09	9.29	7.61	6.43
of Mom rets.	(3.7)	(3.4)	(3.4)	(3.7)	(3.3)	(3.6)	(2.8)	(2.4)	(2.2)	(2.9)	(2.4)	(2.3)

Table 5: Volatility in momentum factor returns on crowding measures

Each column represents a predictive regression of realized volatility in daily momentum factor returns over the next quarter (1981 - 2015) on crowding. Each panel presents three sets of dependent variables using daily: (1) raw returns on the momentum portfolio; (2) residual returns using the Fama and French three factor (FF3) model; and (3) residual on FF3 using dynamic weights. Each set considers the three indicated crowding measures. The regressor 'Crowd' refers to the level of the crowding measure at the end of quarter q-1; $\Delta Crowd_q$ refers to the change over quarter q; and $\hat{\sigma}_{Crowd}$ is the estimate of volatility from a GARCH(1,1) model. 'Realized vol. of Mom rets.' is a control variable equal to the lagged realized volatility of daily momentum returns/residuals over the previous quarter (intercepts not tabulated). The t-statistics are computed with Newey-West standard errors with 3 lags.

Panel A. Crowding measures constructed using four-quarter trading histories									
Dependent variable:	vol of returns			vol of FF3 residuals			vol of dynamic FF3 residuals		
Crowding measure:	Cnt_	CntP1_	Cap_	Cnt_	CntP1_	Cap_	Cnt_	CntP1_	Cap_
$\Delta Crowd_q$	-0.19	-0.45	-0.06	-0.21	-0.44	-0.12	-0.19	-0.38	-0.17
	(-1.1)	(-3.2)	(-0.2)	(-1.2)	(-3.1)	(-0.4)	(-1.0)	(-2.4)	(-0.6)
$Crowd_{q-1}$	-0.14	-0.17	0.05	-0.16	-0.17	0.00	-0.12	-0.15	0.05
	(-2.3)	(-1.9)	(0.4)	(-2.5)	(-2.0)	(0.0)	(-2.2)	(-2.4)	(0.5)
$\hat{\sigma}_{Crowd}$	0.65	1.02	0.68	0.96	1.34	0.86	0.45	0.44	0.51
	(0.8)	(1.2)	(1.9)	(1.1)	(1.5)	(2.3)	(0.8)	(0.7)	(2.0)
Realized vol.	0.76	0.77	0.76	0.72	0.73	0.72	0.73	0.74	0.72
of Mom rets.	(7.4)	(8.1)	(8.1)	(7.1)	(7.9)	(8.2)	(6.7)	(7.5)	(7.6)
Adj-rsquare	63.7%	66.2%	64.2%	60.7%	63.4%	61.7%	59.7%	62.5%	60.7%

Panel B. Crowding measures constructed using one-quarter trading histories									
Dependent variable:	vol of returns			vol of FF3 residuals			vol of dynamic FF3 residuals		
Crowding measure:	Cnt_	CntP1_	Cap_	Cnt_	CntP1_	Cap_	Cnt_	CntP1_	Cap_
$\Delta Crowd_q$	-0.11	-0.08	-0.02	-0.10	-0.08	-0.04	-0.10	-0.08	-0.06
	(-2.8)	(-1.8)	(-0.1)	(-2.9)	(-2.0)	(-0.4)	(-3.4)	(-2.3)	(-0.7)
$Crowd_{q-1}$	-0.07	-0.02	0.04	-0.06	-0.01	0.03	-0.05	-0.03	0.04
	(-1.5)	(-0.3)	(1.3)	(-1.6)	(-0.2)	(0.8)	(-1.6)	(-0.6)	(1.5)
$\hat{\sigma}_{Crowd}$	0.50	1.38	0.10	0.60	1.55	0.16	0.39	1.02	0.08
	(2.3)	(2.6)	(0.6)	(2.7)	(2.9)	(0.9)	(1.9)	(2.1)	(0.7)
Realized vol.	0.78	0.77	0.78	0.75	0.74	0.74	0.75	0.74	0.75
of Mom rets.	(8.3)	(7.8)	(7.3)	(8.5)	(7.9)	(7.1)	(7.6)	(7.1)	(6.9)
Adj-rsquare	66.0%	66.0%	63.2%	63.5%	63.6%	60.0%	63.5%	62.8%	60.0%

Table 6: Momentum factor returns as a determinant of crowding

Each column represents a predictive regression of a different crowding measure on lag momentum returns and lag momentum realized volatility. Panel A shows the results for four-quarter measures and Panel B for one-quarter measures. Volatility is computed using daily momentum returns (intercepts not tabulated). The t-statistics are computed with Newey-West standard errors with 3 lags.

	Panel A			Panel B		
Crowding horizon:	4qtr			1qtr		
Crowding measure:	Cnt_	CntP1_	Cap_	Cnt_	CntP1_	Cap_
1yr return$_{q-1}$	0.23	0.06	0.27	0.47	0.40	0.73
	(1.2)	(0.5)	(2.3)	(2.0)	(2.2)	(2.3)
1yr return$_{q-5}$	0.52	0.48	0.16	0.66	0.68	0.31
	(2.9)	(3.7)	(1.4)	(2.6)	(3.3)	(1.0)
1yr volatility$_{q-1}$	-0.41	-0.31	-0.05	-0.46	-0.37	-0.10
	(-3.3)	(-2.9)	(-0.9)	(-2.9)	(-3.0)	(-0.7)
1yr volatility$_{q-5}$	0.11	0.04	-0.07	0.18	0.07	-0.24
	(0.9)	(0.4)	(-0.9)	(1.3)	(0.5)	(-1.2)
Adj-rsquare	20.0%	17.5%	16.3%	6.5%	6.7%	18.9%

Table 7: Regressions of momentum return moments on crowd count and crowd capital jointly estimated

Each column presents a regression of the indicated momentum return metric as the dependent variable and the indicated horizon for estimating the crowding measure (1qtr or 4qtr). In the case of 'left-tail' the regression is Probit. Return refers to the dynamic Fama-French 3 factor residual for the Probit and Volatility panels, and to a regression with the dynamic FF3 factors as controls in the Returns panel. All regressors in the row headings are included in each regression. Thus, the regressions in the columns labelled 'Returns' correspond to Table 3 using Cnt_ and Cap_ and DFF3 (but estimated jointly). Likewise, the regressions in the columns labelled 'left-tail' and 'Volatility' correspond to jointly estimated versions of Tables 4 and 5, respectively, for the case of Cnt_ and DFF3.

Dependent variable:	returns		10% left-tail		20% left-tail		volatility	
Crowding horizon:	1qtr	4qtr	1qtr	4qtr	1qtr	4qtr	1qtr	4qtr
Crowd = Cnt_:								
$\Delta\text{Crowd}_{\text{q}}$	-0.11	-0.51	4.54	25.4	4.94	23.2	-0.12	-0.13
	(-1.7)	(-3.1)	(1.2)	(2.3)	(1.7)	(2.6)	(-3.2)	(-1.2)
$\text{Crowd}_{\text{q-1}}$	-0.04	-0.32	5.41	15.4	4.18	17.5	-0.07	-0.15
	(-0.5)	(-2.7)	(1.1)	(1.9)	(1.2)	(2.9)	(-2.0)	(-2.7)
$\hat{\sigma}_{\text{Crowd}}$	-0.03	3.03	4.27	3.0	-4.33	-91.9	0.39	0.24
	(-0.1)	(2.2)	(0.2)	(0.1)	(-0.2)	(-1.7)	(2.3)	(0.4)
Crowd = Cap_:								
$\Delta\text{Crowd}_{\text{q}}$	-0.03	0.07	2.42	3.0	-3.25	-11.9	0.07	-0.07
	(-0.3)	(0.3)	(0.4)	(0.2)	(-0.6)	(-0.9)	(0.9)	(-0.3)
$\text{Crowd}_{\text{q-1}}$	0.16	0.74	-4.14	-15.8	-5.20	-24.2	0.05	0.14
	(2.2)	(3.7)	(-1.2)	(-1.4)	(-1.8)	(-2.7)	(1.9)	(1.6)
$\hat{\sigma}_{\text{Crowd}}$	0.06	-0.96	6.79	33.4	6.48	53.3	0.02	0.41
	(0.2)	(-1.4)	(0.7)	(1.4)	(0.7)	(2.4)	(0.2)	(1.7)
Control:								
Realized vol.	-0.27	-0.36	12.35	15.1	7.17	11.0	0.75	0.68
of Mom rets.	(-2.2)	(-3.3)	(3.6)	(3.7)	(2.4)	(3.1)	(6.9)	(6.2)